On multivariate geometric random sums

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We can use multivariate geometric distribution to generalize the notion of geometric random sum to the multidimensional case.

To date have been studied limit distributions, which approximate the geometric sums in the form $\sum\limits_{j=1}^{L} W^{(j)}$, where $W^{(j)} = \left(W_1^{(j)},\, \ldots,\, W_k^{(j)}\right)$ are independent identically distributed k-dimensional random vectors, L is a random variable having geometric distribution; L and $W^{(j)}$ $(j = 1,\, 2,\, \ldots)$ are independent. Note that the number of terms will be the same for each component.

Let us consider the more general case. The number of random variables L_j $(j = 1,\, \ldots,\, k)$ will be different for each component, while values of L_j could be **dependent**.

Multivariate geometric random sum is called a random vector sum of the form $S = (S_1, \ldots, S_k) = \left(\sum\limits_{j=1}^{L_1} W_1^{(j)}, \ldots, \sum\limits_{j=1}^{L_k} W_k^{(j)}\right)$, where L_m $(m = 1, \ldots, k)$ will be defined below, $W_m^{(j)}$ are independent random variables identically distributed for each m having known characteristic function $E\, exp(i\, t_m\, W_m) = \varphi_m(t_m)$; lastly L_m and $W_m^{(j)}$ are independent.

The vector $L = (L_1, \ldots, L_k)$ is introduced by the following way. Let $\mathcal{E} = \{\epsilon\}$ be a set of $k-$dimensional indices $\epsilon = (\varepsilon_1,\, \ldots,\, \varepsilon_k)$ and each component of ε_m is 0 or 1; $\mathcal{E}_m$ is a set of $k-$dimensional indices for which $\varepsilon_m = 1$; N_ε are independent geometrically distributed random variables with parameters p_ε. By definition, put the value $L_m = \min\limits_{\varepsilon \in \mathcal{E}_m} \{N_\varepsilon\}$.

We show that the limit distributions of such sums by the corresponding normalization can be:

- multivariate exponential distribution introduced by Marshall and Olkin;
- multivariate generalized Laplace distribution introduced earlier by author.

Let us define the *marginally strictly geometric stable distribution* as the distribution of vector $R = (Z_1^{1/\alpha_1} Y_1, Z_2^{1/\alpha_2} Y_2, \ldots, Z_k^{1/\alpha_k} Y_k)$, where Y_m are independent random variables with strictly stable distributions with characteristic functions $g_m(\theta_m)$ and parameters $\alpha_m, \eta_m, \beta_m$; $Z = (Z_1, \ldots, Z_k)$ is independent from $Y_1, ..., Y_k$, random vector having the Marshall-Olkin multivariate exponential distribution.

Now let $p_\varepsilon = \lambda_\varepsilon p$. Assume that $\varphi_m(p^{1/\alpha_m}\theta_m) = 1 + p \ln g_m(\theta_m) + o(p)$ as $p \to 0$. We have proved that the normalized vector $\left(p^{1/\alpha_1} \sum\limits_{j=1}^{L_1} W_j^{(1)}, \ldots, p^{1/\alpha_k} \sum\limits_{j=1}^{L_k} W_j^{(k)}\right)$ converges weakly to R as $p \to 0$, where R has the *marginally strictly geometric stable distribution*.