On a family of robust estimators for autocorrelation coefficients under outliers

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We consider observations $\{y_t\}$ derived from Gaussian stationary time series $\{x_t\}$, distorted by the so-called replacement outliers [3]. The robust ψ-estimate $\hat{\theta}_\tau$ for autocorrelation coefficient $\theta_\tau = \mathbb{CORR}\{x_t, x_{t+\tau}\}$ of undistorted (hidden) time series $\{x_t\}$ is computed from observed distorted time series $\{y_t\}$ by the formula [1]:

$$\hat{\theta}_\tau ::= f_\psi^{-1}\left(\frac{(1-\varepsilon)^{-2}}{T-\tau}\sum_{t=1}^{T-\tau}\psi\left(\frac{y_t}{y_{t+\tau}}\right)\right),\ 0<\tau<T, \tag{1}$$

where $\psi : \mathbb{R} \to \mathbb{R}$ is an odd bounded function, $f_\psi(\theta) ::= \mathbb{E}\psi(\zeta)$, ζ is the Cauchy distributed random variable with law $\mathcal{C}(\theta, \sqrt{1-\theta^2})$, $0 < \varepsilon \ll 1$ is the probability of a replacement outlier presence in $\{y_t\}_{t=1}^T$.

Under several assumptions on function ψ and asymptotical behavior of autocorrelation θ_τ at $\tau \to \infty$ the ψ-estimator (1) is shown to be consistent and asymptotically Gaussian [1, 2]. Some examples of $\psi(\cdot)$ generated ψ-estimators in the family (1) are given. Optimal function $\psi_*(\cdot)$ that minimizes the functional (an approximation for the mean squared error for the ψ-estimator) is found. Numerical comparison of ψ-estimator (1) w.r.t. the robust Huber estimator [4] is made based on real and simulated data.

References

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