On reduced rank regression analysis in GMANOVA-MANOVA models

Tatjana von Rosen[1] and Dietrich von Rosen[2,3]

[1]*Stockholm University, Sweden, tatjana.vonrosen@stat.su.se*
[2]*Swedish University of Agricultural Sciences, Sweden, Dietrich.von.Rosen@slu.se*
[3]*Linköping University, Sweden*

Keywords: growth curve model, maximum likelihood estimator, multivariate analysis of variance, reduced rank model

We discuss the maximum likelihood estimation in general multivariate linear models where the rank restrictions are imposed on the matrix of regression coefficients in order to enable parsimonious modeling. In particular, the following GMANOVA-MANOVA model is of interest:

$$\boldsymbol{X} = \boldsymbol{A}\boldsymbol{B}_1\boldsymbol{C}_1 + \boldsymbol{B}_2\boldsymbol{C}_2 + \boldsymbol{E},$$

where the columns of $\boldsymbol{X}$: $p \times n$ are p-dimensional independent normally distributed response vectors, $\boldsymbol{A}: p \times q_1$, is a known within-individuals design matrix, $\boldsymbol{C}_1$: $k_1 \times n$ is a known between-individuals design matrix, $\boldsymbol{B}_1 : q_1 \times k_1$ is an unknown parameter matrix that summarizes the mean structure of the response variables over time, $\boldsymbol{B}_2$: $p \times k_2$ describes the mean structure, $\boldsymbol{C}_2$: $k_2 \times n$ is a known between-individuals design matrix, the columns of $\boldsymbol{E}$: $p \times n$ are independent normally distributed random error vectors, $\boldsymbol{E}_k \sim N_p(0, \Sigma)$, where $k = 1, \ldots, n$, and Σ is an unknown positive definite matrix.

This model being the mixture of the growth curve (GMANOVA) and the multivariate analysis of variance (MANOVA) models extends the classical growth curve model to include the covariates. The GMANOVA-MANOVA model was proposed by Chinchilli & Elswick (1985) and von Rosen (1989), and has been applied in a variety of disciplines, including economics, biology, medicine, engineering and life sciences.

Several modifications of the GMANOVA-MANOVA model will be studied depending on the rank restrictions which are characterized by the following conditions:
(i) $r(\boldsymbol{B}_2) = q_2 < \min(p, k_2)$, $q_1 < p$, $C(\boldsymbol{C}_2')C(\boldsymbol{C}_1')$;
(ii) $r(\boldsymbol{B}_2) = q_2 < \min(p, k_2)$, $\boldsymbol{A} = \boldsymbol{I}_p$;
(iii) $r(\boldsymbol{B}_2) = q_2 < \min(p, k_2)$, $q_1 < p$, $C(\boldsymbol{C}_1') \subseteq C(\boldsymbol{C}_2')$;
(iv) $r(\boldsymbol{B}_1) = f < \min(q_1, k_1)$, $r(\boldsymbol{B}_2) = q_2 < \min(p, k_2)$, $C(\boldsymbol{C}_2') \subseteq C(\boldsymbol{C}_1')$,
where $C(\bullet)$ denotes the column vector space.

References

[1] Chinchilli, V.M. and Elswick, R.K. (1985). A mixture of the MANOVA and GMANOVA models. *Comm. Statist. Theory Methods* **14**, 3075–3089.

[2] von Rosen, D. (1989). Maximum likelihood estimators in multivariate linear normal models. *Journal of Multivariate Analysis* **31**, 187–200.