The existence of infinite Viterbi alignment for PMC models

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We consider a two-dimensional homogeneous Markov chain $((X_1, Y_1), (X_2, Y_2), ...)$, where random variables X_i (observations) are taking values from some set $\mathcal{X}$ and random variables Y_i (unobserved or "hidden" states) are taking values from state space $\mathcal{Y} = \{1, ..., |\mathcal{Y}|\}$. Following Pieczynski [1] we call this model a pairwise Markov chain (PMC). The name reflects the fact that conditionally, given the marginal process $X = (X_1, X_2, ...)$, the process $Y = (Y_1, Y_2, ...)$ is a Markov chain, and conditionally, given Y, X is a Markov chain. In general though, neither X nor Y are necessarily Markov chains. PMC is a natural generalization of hidden Markov model (HMM). In particular, just like in case of HMM, given a realization $x_{1:n} = (x_1, \ldots, x_n)$ of $X_{1:n} = (X_1, \ldots, X_n)$, the Viterbi algorithm can be employed to find the maximum a posteriori (MAP) estimate $(v_1(x_{1:n}), ..., v_n(x_{1:n}))$ of $Y_{1:n}$. This estimate is also called the Viterbi alignment or Viterbi path. We prove a theorem which gives sufficient conditions for extending the Viterbi alignment to infinity. We further provide examples where we apply this theorem to specific models.

References

[1] Pieczynski, W. (2003). Pairwise Markov chains. *IEEE Transactions on Pattern Analysis and Machine Intelligence* **25**, 634–639.