

# A Lundberg-type inequality for an inhomogeneous renewal risk model

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We say that the insurer's surplus  $U(t)$  varies according to the inhomogeneous renewal risk model if

$$U(t) = x + ct - \sum_{i=1}^{\Theta(t)} Z_i, t \geq 0,$$

where  $x \geq 0$  is the initial risk reserve;  $c > 0$  is the constant premium rate;  $\{Z_1, Z_2, \dots\}$  are independent non-negative claim sizes;  $\Theta(t)$  is the number of accidents in the interval  $[0, t]$  given by formula

$$\Theta(t) = \sum_{n=1}^{\infty} \mathbf{I}_{\{\theta_1 + \theta_2 + \dots + \theta_n \leq t\}}$$

and  $\{\theta_1, \theta_2, \dots\}$  are non-negative and non-degenerate at zero random variables, standing for the inter-arrival times. In addition, we suppose that sequences  $\{Z_1, Z_2, \dots\}$  and  $\{\theta_1, \theta_2, \dots\}$  are mutually independent.

If all claim sizes  $\{Z_1, Z_2, \dots\}$  are identically distributed and all inter-arrival times  $\{\theta_1, \theta_2, \dots\}$  are also identically distributed, then the inhomogeneous renewal risk model becomes the homogeneous renewal risk model.

The time of ruin and the ruin probability are the main critical characteristics of the homogeneous and the inhomogeneous risk models. The first time  $\tau$  when the surplus  $U(t)$  drops to a level less than zero is called the time of ruin. In such a case, the ruin probability  $\psi$  is defined by equality  $\psi(x) = \mathbb{P}(\tau = \infty)$ , where  $x$  is the initial reserve which is supposed to be the main model parameter.

The Lundberg inequality is well known for the homogeneous renewal risk model. This inequality states that  $\psi(x) \leq e^{-Hx}$  for some positive  $H$  in the case when  $\mathbb{E}Z_1 - c\mathbb{E}\theta_1 < 0$  and  $\mathbb{E}e^{hZ_1} < \infty$  for some positive  $h$ . The proofs of this statement can be found for instance in [2] or [3].

One can show that the similar exponential estimate of the ruin probability holds for an inhomogeneous risk renewal model. Naturally, we need for such estimate to use more complex requirements for random variables  $\{Z_1, Z_2, \dots\}$  and  $\{\theta_1, \theta_2, \dots\}$ . The exact formulation of the Lundberg-type inequality for an inhomogeneous renewal risk model together with the detailed proof can be found in [1].

## References

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