Tail probabilities of likelihood ratio statistic

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Let $\mathbf{y} = (y_1, \ldots, y_n)$ be a random vector having the multinomial distribution

$$\mathbf{y} \sim \text{Multinomial}_n(N, \mathbf{p}),$$

where $\mathbf{p} = (p_1, \ldots, p_n)$ is a vector of positive probabilities.

Define (logarithmic) likelihood ratio statistic

$$G_n^2(\mathbf{y}; N, \mathbf{p}) := \sum_{i=1}^{n} y_i \log \left(\frac{y_i}{p_i N} \right).$$

Assuming that $p_{min} := \min_{i=1,\ldots,n} p_i \geq \delta_0 > 0$, Hoeffding [1] proved that

$$\mathbf{P}\{G_n^2(\mathbf{y}; N, \mathbf{p}) \geq x\} = O\left(x^{(n-3)/2} \mathrm{e}^{-x}\right), \quad N \to \infty,$$

uniformly in $x \in [c_1, c_2 N]$ for arbitrary positive constants c_1 and c_2. Kallenberg [2] obtained upper and lower bounds for the tail probabilities of $G_n^2(\mathbf{y}; N, \mathbf{p})$ in the case where $p_{min} \to 0$ and $n \to \infty$ not too fast. However, the upper bound exceeds the corresponding lower bound by a factor of order $\sqrt{x}$.

The *problem* is to obtain, for the tail probabilities of $G_n^2(\mathbf{y}; N, \mathbf{p})$, an exact (up to a constant factor) upper bound valid for all positive x.

For $n = 2$, the universal and exact up to a factor of 2 upper bound for the tail probabilities follows from the paper by Zubkov and Serov [3]. The generalization of this bound to the case $n > 2$ is discussed.

References

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