Gerber-Shiu discounted penalty function for the bi-seasonal discrete time risk model

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The bi-seasonal discrete time risk model for insurer's surplus (property) changing over the time $n \in \mathbb{N}_0 := 0, 1, 2, \ldots$ is described by the following equation:

$$U_u(n) = u + n - \sum_{i=1}^{n} Z_i,$$

where $u = U_u(0) \in \mathbb{N}_0$, is the insurer's (insurance company's) initial surplus; Z_i, $i \in N$ are claim amounts which assumed to be independent nonnegative integer-valued random variables satisfying the following conditions:

$$Z_{2k+1} \stackrel{d}{=} Z_1, \ Z_{2k+2} \stackrel{d}{=} Z_2, \ k = 0, 1, 2, \ldots.$$

If $Z_1 = Z_2$, then the bi-seasonal discrete time risk model reduces to the classical discrete time risk model.

The finite time ruin probability $\psi(u, t)$, the ruin probability $\psi(u)$ and Gerber-Shiu discounted penalty function $\Psi_\delta(u)$ are the main extremal characteristics for both models.

Gerber-Shiu discounted penalty function for the model is defined by equality:

$$\Psi_\delta(u) = \mathbb{E}^{-\delta T_u} \mathbb{1}_{\{T_u < \infty\}},$$

where $\delta \geqslant 0$, $u \in \mathbb{N}_0, T_u$ – the time of ruin, i.e.

$$T_u = \begin{cases} \min \ \{n \geqslant 1 : U_u(n) \leqslant 0\}, \\ \infty, \text{ if } U_u(n) > 0 \text{ for all } n = 1, 2, 3, \ldots. \end{cases}$$

Furthermore

$$\psi(u) = \Psi_0(u) = \mathbb{E}\mathbb{1}_{\{T_u < \infty\}} = \mathbb{P}\{T_u < \infty\}, \psi(u, t) = \mathbb{P}\{T_u \leqslant \infty\}.$$

The formula for calculations of the finite time ruin probability $\psi(u, t)$, the ruin probability $\psi(u)$ and Gerber-Shiu discounted penalty function $\Psi_\delta(u)$ for the classical discrete time risk model are presented in [3].

The formula for calculation of $\psi(u, t)$ is given in the article [1].

The formula for calculation of $\psi(u)$ is presented in [2].

The similar formula for the bi-seasonal discrete time risk model and for Gerber-Shiu discounted penalty function $\Psi_\delta(u)$ can be obtained.

References

[1] Blaževičius, K., Bieliauskienė, E., Šiaulys, J. (2010). Finite-time ruin probability in the inhomogeneous claim case. *Lithuanian Mathematical Journal* **50**, 260–270.

[2] Damarackas, J. and Šiaulys, J. (2014). Bi-seasonal discrete time risk model. *Applied Mathematics and Computation* **247**, 930–940.

[3] Dickson, D.C.M. (2010). *Insurance Risk and Ruin*. Cambridge University Press, Cambridge.