Semiparametric density estimation for star-shaped distributions

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In the talk we discuss properties of semiparametric estimators for the density generator function, and for the density in a star-shaped distribution model. The approach under consideration modifies and generalizes that of an earlier paper [1] about elliptical distributions. The semiparametric procedure combines the flexibility of nonparametric estimators and the simple estimation and interpretation of parametric estimators. A parametric model is assumed for the density contours given by the star body. The parameters are estimated using a moment estimation method. The star generalized radius density is estimated nonparametrically by use of a kernel density estimator. Since the star generalized radius density is a univariate function, we avoid the disadvantages of nonparametric estimators in connection with the curse of dimensionality.

The density of the star-shaped distribution of the random vector X is given by

$$\varphi_{g,K,\mu}(x) = C(g,K)\ g\left(h_K(x-\mu)\right) \text{ for } x \in \mathbb{R}^d,$$

where $C(g,K)$ is the normalizing constant, g is the density generator function, K is the star body and h_K the corresponding Minkowski functional. The theory of star-shaped distributions is developed in [2]. We suppose that a formula for h_K is specified. If h_K involves some additional parameters, then estimators of them are to be provided. In the paper moment estimators are used. Let $\psi : [0,\infty) \to \mathbf{R}$ be a strictly increasing function. The semiparametric estimator can be calculated by

$$\hat{\varphi}_n(x) = C(g,K)\ \hat{g}_n\left(h_K(x-\hat{\mu}_n)\right) \text{ for } x \in \mathbb{R}^d,$$

where

$$\hat{g}_n(z) = z^{1-d}\ \psi'(z)\ \hat{\chi}_n(\psi(z)) \text{ for } z \in \mathbb{R},$$

and $\hat{\mu}_n$ is the average of all sample items. In the last formula $\hat{\chi}_n$ is a kernel estimator for the density of $\psi(h_K(X-\mu))$.

In the talk, results on convergence rates of the density estimator are presented. It turns out that, in the case where a neighbourhood of the center μ is excluded, these rates coincide with the rates known from usual one-dimensional kernel density estimators. The behaviour of the estimator in the center of the distribution is discussed, too. We show that the density estimator is asymptotically normally distributed.

References

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