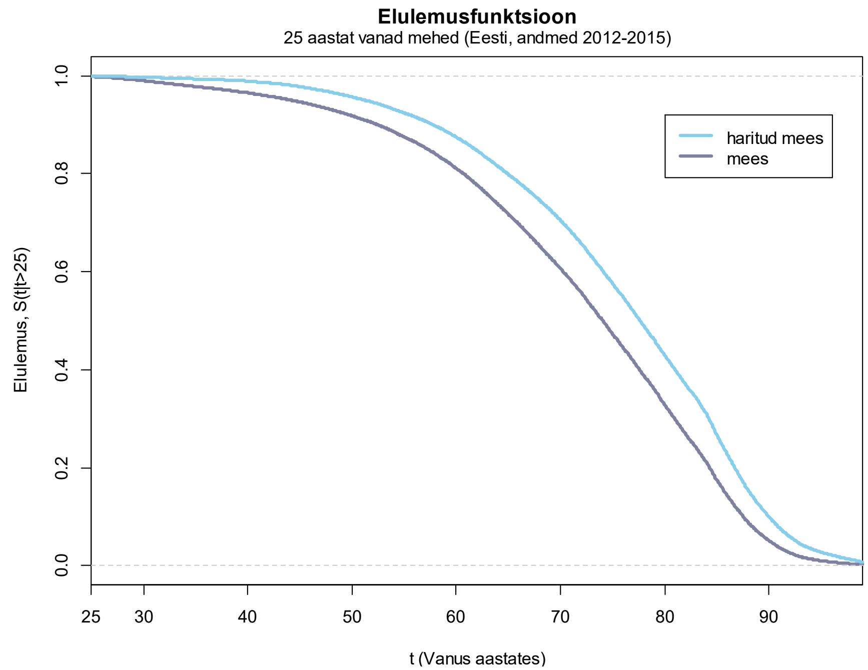
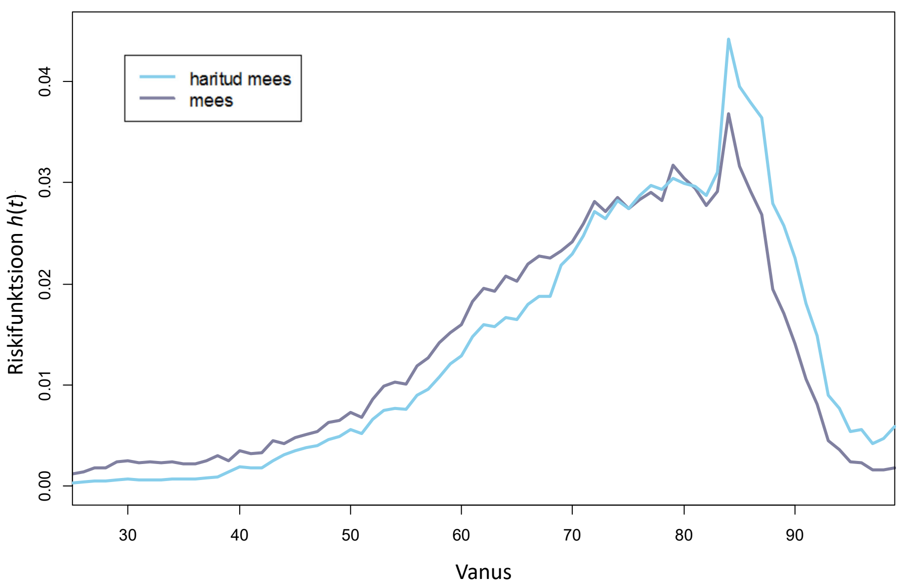
Elukestvusanalüüs II

Riskifunktsioon $h(t)$





Lisaks...

T — eluiga (aeg teatud sündmuseni)

$S(t) := P(T > t)$ elukestvusfunktsioon

$$h(t) := \lim_{\Delta t \to 0+} \frac{P(t < T \leq t + \Delta t | T > t)}{\Delta t}$$ riskifunktsioon

Kumulatiivne riskifunktsioon

$$H(t) := \int_0^t h(u) du$$

Seoseid

$$h(t) = \frac{f(t)}{S(t)}$$
$$= -(\ln S(t))'$$

$$H(t) = -\ln S(t)$$

$$S(t) = \exp(-H(t))$$

$$f(t) = h(t)S(t)$$

$$E(T) = \int_0^\infty S(t)dt$$

Seoseid

$$h(t) = \frac{f(t)}{S(t)}$$
$$= -(\ln S(t))'$$

$$H(t) = -\ln S(t)$$

$$S(t) = \exp(-H(t))$$

$$f(t) = h(t)S(t)$$

$$E(T) = \int_0^\infty S(t)dt$$

$$\begin{aligned} h(t) &:= \lim_{\Delta t \to 0+} \frac{P(t < T \le t + \Delta t | T > t)}{\Delta t} \\ &= \lim_{\Delta t \to 0+} \frac{P(t < T \le t + \Delta t \bigcap T > t)}{P(T > t)} \frac{1}{\Delta t} \\ &= \lim_{\Delta t \to 0+} \frac{P(t < T \le t + \Delta t)}{P(T > t)} \frac{1}{\Delta t} \\ &= \lim_{\Delta t \to 0+} \frac{F(t + \Delta t) - F(t)}{\Delta t} \frac{1}{P(T > t)} \\ &= f(t) \frac{1}{P(T > t)} \\ &= \frac{f(t)}{S(t)} \end{aligned}$$

$$E(T) = \int_0^\infty S(t)\,\mathrm{d}t$$

$$T = \int_0^\infty I(T > u)\,\mathrm{d}u$$

$$E(T) = \int_0^\infty t f(t)\,\mathrm{d}t$$

$$\begin{aligned} E\left(\int_0^\infty I(T > u)du\right) &= \int_0^\infty \int_0^\infty I(t > u)\,\mathrm{d}u f(t)\,\mathrm{d}t \\ &= \int_0^\infty \int_0^\infty I(t > u) f(t)\,\mathrm{d}t\,\mathrm{d}u \\ &= \int_0^\infty \left(\int_0^u 0 f(t)\mathrm{d}t + \int_u^\infty 1 f(t)\mathrm{d}t\right)\,\mathrm{d}u \\ &= \int_0^\infty (0 + F(\infty) - F(u))\,\mathrm{d}u \\ &= \int_0^\infty S(u)\,\mathrm{d}u \end{aligned}$$

Mudelid riskile

- Eksponentjaotus, $T \sim \mathrm{Exp}(\lambda)$

$$h(t) = \frac{f(t)}{S(t)} = \frac{\lambda e^{-\lambda t}}{e^{-\lambda t}} = \lambda$$

$$f(t) = \lambda e^{-\lambda t}$$
$$F(t) = 1 - e^{-\lambda t}$$
$$S(t) = e^{-\lambda t}$$

Mudelid riskile

- Eksponentjaotus, T~Exp(λ)

$$h(t) = \frac{f(t)}{S(t)} = \frac{\lambda e^{-\lambda t}}{e^{-\lambda t}} = \lambda$$

- Weibull, T~Wei(λ; p) $h(t) = \lambda^p p t^{p-1}$
- Gompertz-Makeham $h(t) = e^{\alpha + \beta t}$
- Gamma ilus valem puudub
- Generalized Gamma ilus valem puudub
- log-Normal

Hindamine tsenseeritud vaatluste puhul

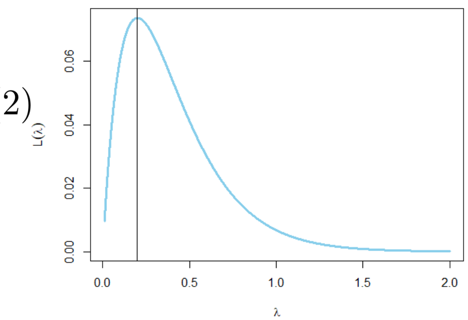
- Eksponentjaotus, T~Exp(λ)

$$f(t) = \lambda e^{-\lambda t}$$
$$F(t) = 1 - e^{-\lambda t}$$
$$S(t) = e^{-\lambda t}$$

$$h(t) = \frac{f(t)}{S(t)} = \frac{\lambda e^{-\lambda t}}{e^{-\lambda t}} = \lambda$$

Vaatlused: 1 2+ 2+

$$L(\lambda) = f(1) \cdot S(2) \cdot S(2)$$
$$\hat{\lambda} = 0,2$$
$$\widehat{\mathrm{E}(T)} = \frac{1}{\hat{\lambda}} = \frac{1}{0,2} = 5$$



Mudelid riskile – mitteparameetriline lähenemine

- Milline ikkagi on eluigade T jaotus?
- Kas me tingimata peame seda jaotust teadma?
- Cox'i võrdeliste riskide mudel:

naised $h_0(t)$ $h(t) = h_0(t)\exp(\beta_1 \cdot 0)$

mehed $h_0(t)c_{mees}$ $h(t) = h_0(t)\exp(\beta_1 \cdot 1)$

$$h(t) = h_0(t)\exp(\beta_1 x_1 + \beta_2 x_2 + \ldots)$$

$$x_1 = \begin{cases} 0, & \text{naine} \\ 1, & \text{mees} \end{cases}$$

Cox'i võrdeliste riskide mudel

$$h(t|x_1, x_2, \ldots) = h_0(t)\exp(b_1 x_1 + b_2 x_2 + \ldots)$$

```
andmed <- data.frame(
              time=c(4,3,1,1,2,2,3),
              status=c(1,1,1,0,1,1,0),
              sugu=c(0,0,0,0,1,1,1))

mudel = coxph(Surv(time, status) ~ factor(sugu), andmed)
summary(mudel)
```

```
andmed <- data.frame(
               time=c(4,3,1,1,2,2,3),
               status=c(1,1,1,0,1,1,0),
               sugu=c(0,0,0,0,1,1,1))

mudel = coxph(Surv(time, status) ~ factor(sugu), andmed)
summary(mudel)
```

```
[...]

  n= 7, number of events= 5

                coef exp(coef) se(coef)     z Pr(>|z|)
factor(sugu)1 0.1438    1.1547   1.0198 0.141    0.888

              exp(coef) exp(-coef) lower .95 upper .95
factor(sugu)1     1.155      0.866    0.1565     8.522
```