# Modeling of replicated response measures by using interval arithmetic

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In experimental design, some data sets are composed with the replicated response measures. The replicated response measures are assumed to have uncertainty in the quantification of their values. In this case, it may be suitable to represent these values by using fuzzy numbers. The fuzzy numbers are an extension of real numbers which allow the incorporation of uncertainty [1].

In this study, the replicated measures of the responses are represented as triangular type-1 fuzzy numbers similar to [2]. In addition, trapezoidal and Gaussian type-1 fuzzy numbers are used for comparison purposes. The novelty of this study is the formalization of these fuzzy numbers as intervals. In order to convert the fuzzy numbers to intervals, $\alpha$-cut values of fuzzy numbers are used. Here, $\alpha$ is chosen equal to 0 ($\alpha = 0$) for the simplicity. It should be noted here that the responses and model parameters are assumed to be interval values whereas inputs are crisp. Interval arithmetic is used to estimate the unknown model parameters in which the least squares criterion is applied. Since the interval matrix multiplication is a major time consuming process, midpoint and radius values are used for calculations instead of min-max operators as in [3], [4]. Then the objective function is expressed as an interval where the upper and lower bounds need to be minimized simultaneously. By considering each bound as an objective function, the estimation problem is solved by using Nondominated-Sorting Genetic Algorithm-II (NSGA–II) which is a well-known multi objective optimization method [5]. In the application part, a well-known data set, the wheel cover component data set [6] is used. Finally, the conclusions and future research directions are given in the conclusion section.

## References

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