Multicomponent stress-strength reliability for a multivariate Weibull distribution

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Keywords: multivariate Weibull distribution, stress-strength model, s-out-of-$k : G$ system

An s-out-of-$k : G$ system consists of k component functions if and only if at least s components function. The s-out-of-$k : G$ system has wide applications in both industrial and military systems. For example, an automobile with four tires is usually equipped with one additional spare tire. Hence, the vehicle can be driven as long as at least 4-out-of-5 tires are in good condition. For an extensive reviews of s-out-of-k and related systems, see [3].

In this study, we consider the s-out-of-$k : G$ system which has k statistically independent and identically distributed strength components (or subsystems) each consisting of m statistically dependent elements. The system is subjected to a common random stress and works if at least s $(1 \leq s \leq k)$ components simultaneously operate. In that case, the strength component (or subsystem) is alive only if the weakest element is operating, namely it is regarded as a series system.

We assume that each strength component, namely $(X_{i1}, X_{i2}, ..., X_{im})$, $i = 1, ..., k$, follows a multivariate Weibull (MVW) distribution (see [2] and [4]) and a common random stress variable T follows a Weibull distribution. Similar system is considered in [5] when the strength components consist of a pair of statistically dependent elements. Let $Z_i = \min(X_{i1}, X_{i2}, ..., X_{im})$, $i = 1, ..., k$. In terms of these random variables, the system is working if at least s $(1 \leq s \leq k)$ of the Z_i strength variables operate when the common stress variable T is carried out. The reliability of this multicomponent stress-strength model is given by $R_{s,k} = P(\text{at least } s \text{ of the } (Z_1, ..., Z_k) \text{ exceed } T)$ (see [1]). In this study, the estimates of $R_{s,k}$ are investigated by using classical and Bayesian approaches. Then, the derived estimates are compared through Monte Carlo simulations.

References

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