Markov-modulated multivariate linear regression

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We consider the case, where a process, described by multivariate linear regression, operates in a random environment. The last is presented as a continuous-time homogeneous irreducible Markov chain $J(t), t \geq 0$, with finite state set $N = \{1, 2, ..., k\}$ [1]. Let $\lambda_{i,j}$ be the known transition rate from state i to state $j\big(\lambda_{j,j} = 0\big)$.

The following notations will be used for the η-th observation ($\eta = 1, ..., n$): $x_{(\eta)} = (x_{\eta,1}, ..., x_{\eta,q})$ is the q-row vector of known independent variables; $Y_{(\eta)}(t) = (Y_{\eta,1}(t), ..., Y_{\eta,p}(t))$ is the p-row vector of observed dependent variables; $e_{(\eta)} = (e_{\eta,1}, ..., e_{\eta,p})$ is the p-row vector of random variables, $e_{(\eta)} \in N_p(0, I)$; t_η is the observation time; $T_{\eta,\mu}$ is an unobserved sojourn time in the state $\mu \in N (T_{\eta,1} + ... + T_{\eta,k} = t_\eta)$. Further the $q \times p$-matrix $B(j)$ of regression parameters for the j-th state of the random environment ($j = 1, ..., k$) and the symmetric square root $\sum^{1/2}$ of the positive definite matrix $\sum$ are unknown and identical for all observations.

Thus, if $T_{(\eta)} = (T_{\eta,1}, ..., T_{\eta,p})$ and $\tilde{B} = (B(1)^T, ..., B(k)^T)^T$, then we have the model for the η-th observation:

$$Y_{(\eta)}(t_\eta) = \big(T_{(\eta)} \otimes x_{(\eta)}\big)\tilde{B} + \sqrt{t_\eta} e_{(\eta)}, \quad \eta = 1, ..., n.$$

We consider estimators of $\tilde{B}$ and $\sum$ for the following given data on n observations: the vectors $Y_{(\eta)}$ and $x_{(\eta)}$ of dependent and independent variables; observation time t_η; the initial $i_\eta = J(0)$ and finite $j_\eta = J(t_\eta)$ states of Markov chain. It is supposed that all observations are independent.

Obtained results generalize the previous results of the author for the multiple linear regression [2].

References

[1] Pacheco, A., Tang, L.C., Prabhu, N.U. (2009). *Markov-Modulated Processes & Semiregenerative Phenomena*. World Scientific, New Jersey, London.

[2] Andronov, A. (2012). *Parameter statistical estimates of Markov-modulated linear regression.* In: Statistical Methods of Parameter Estimation and Hypothesis Testing **24**, Perm State University, Perm, Russia, 163–180 (in Russian).