

On elliptical multivariate quantiles

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Inspired by nonlinear quantile regression, we propose a concept of elliptical location quantiles in regression setup. For m -variate response \mathbf{Y} and p -variate regressor \mathbf{Z} we define elliptical quantile region as

$$\mathcal{E}_\tau(\mathbf{Y}, \mathbf{Z}) = \{(y, z); (y - s_\tau(z))^T A_\tau(z)(y - s_\tau(z)) < c_\tau(z)\},$$

where $s_\tau(z) \in \mathbb{R}^m$ (the centre of the ellipsoid), $A_\tau(z)$ is $m \times m$ (non-singular matrix with unit determinant), and $c_\tau(z)$ are functions of the regressor such that the overall coverage of the elliptical quantile region is τ .

The elliptical quantiles are affine equivariant and may be computed quite efficiently even for large datasets. It is also possible to cover quite general trends and various forms of heteroscedasticity. Therefore the elliptical quantiles may be a good option for testing trend or symmetry of the response variables. It is also possible to incorporate many types of a priori information regarding the model parameters.

References

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