Caïssan squares: the magic of chess

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We study various properties of $n \times n$ Caïssan magic squares. A magic square is Caïssan whenever it is pandiagonal and knight-Nasik, so that all paths of length n by a chess bishop are magic (pandiagonal) and by a (regular) chess knight are magic (CSP2-magic).

Following the seminal 1881 article [4] by "Ursus" in *The Queen,* we show that 4-ply magic matrices, or equivalently magic matrices with the "alternate-couplets" property, have rank at most equal to 3. We also show that an $n \times n$ magic matrix $\mathbf{M}$ with rank 3 and index 1 is EP if and only if $\mathbf{M}^2$ is symmetric. We identify and study 46080 Caïssan beauties—Caïssan magic squares which are also CSP3-magic; a CSP3-path is made by a special knight that leaps over 3 instead of 2 squares. We find that just 192 of these Caïssan beauties are EP. We generalize an algorithm given by Cavendish [2:(1894)] for generating Caïssan beauties and find these are all EP. We also study the n-queens problem first posed with $n = 8$ by Bezzel [1:(1848)] and the Firth–Zukertort "magic chess board" due to Firth [3:(1887)].

An extensive annotated and illustrated bibliography of over 300 items, many with hyperlinks, ends our report. We give special attention to items by (or connected with) "Ursus": Henry James Kesson (b. c. 1844), Andrew Hollingworth Frost (1819–1907), Charles Planck (1856–1935), and Pavle Bidev (1912–1988). We have tried to illustrate our findings as much as possible, and whenever feasible with images of postage stamps or other philatelic items.

References

[1] [Max Friedrich Wilhelm Bezzel (1824–1871)] (1848). Vor einige Zeit wurden uns von einem Schachfreunde zwei Fragen vorgelegt .... *Schachzeitung der Berliner Schachgesellschaft* **3**, p. 363.

[2] "Cavendish" [Henry Jones (1831–1899)] (1894). *Recreations with Magic Squares: the eight queens' problem solved by magic squares and domino squares,* Thomas de la Rue, London.

[3] Firth, W. A. [William A. Firth (c. 1815–1890)] (1887). *The Magic Square,* printed by R. Carswell & Son, Belfast.

[4] "Ursus" [Henry James Kesson (b. c. 1844)] (1881). Caïssan magic squares. *The Queen: The Lady's Newspaper & Court Chronicle,* **70**, p. 142 (August 6, 1881), pp. 276–277 (September 10, 1881) & p. 391 (October 15, 1881).