Depth-based classification for functional data

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Keywords: band depth, classification, data depth, functional data.

The nonparametric classification of data from a subspace of continuous functions $C([0,1])$ will be discussed. Special attention will be paid to depth-based classification rule and its possible generalizations. The decision rule is related to the concept of data depth, which is in this case a functional

$$D : C([0,1]) \to [0,1].$$

Depth is a measure of centrality of an observation with respect to a data set or a distribution. Recently several authors proposed their notions of depth for functional data (Fraiman and Muniz [2], López-Pintado and Romo [5]). These depth functionals are invariant with respect to a domain permutation

$$T : C([0,1]) \to C([0,1]) : x(t) \mapsto x(\phi(t)),$$

where ϕ is a bijection of $[0,1]$ and $t \in [0,1]$. Thus, none of the established depth functionals is able do deal with the shape of functions.

This problem will be demonstrated in a functional classification task. A new class of depth functionals, K-band depths for $K \in \mathbb{N}$ will be utilized in order to handle it. The simplicial depth described by Liu [4] along with Fraiman-Muniz method are employed to involve derivatives into depth computation. The performance of the new approach is compared to similar results obtained by Cuveas et al. [1] in a simulation study of functional data supervised classification. We show that proper derivative using in combination with DD-plot (Depth-Depth plot) techniques proposed by Li et al. [3] is a powerful tool not only for the classification of functional observations.

References

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