The asymptotic results for nearly critical branching processes with immigration

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Let $\{\xi_{k,j}^{(n)},\ k,j \in \mathbb{N}\}$ and $\{\epsilon_k^{(n)},\ k \in \mathbb{N}\}$ be two independent sequences of non-negative integer-valued and identically distributed random variables for every $n \in \mathbb{N}$. For $n \in \mathbb{N}$ we define a sequence of random variables recursively:

$$X_0^n = 0, \quad X_k^n = \sum_{j=1}^{X_{k-1}^n} \xi_{k,j}^{(n)} + \epsilon_k^{(n)},\ k \in \mathbb{N}.$$

The sequence $\{X_k^n\ k \in \mathbb{N}\}$ is called a branching process with immigration [1]. We assume that $m_n = \mathbb{E}(\xi_{1,1}^{(n)})^2 < \infty$ and $\mathbb{E}(\epsilon_1^{(n)})^2 < \infty$ for all $n \in \mathbb{N}$. The branching process with immigration is called nearly critical if $m_n \to 1$ as $n \to \infty$.

In the papers [2]–[4] asymptotic behavior of the process $X_{[nt]}^n,\ t > 0$ has been investigated in the case $m_n = 1 + \alpha d_n^{-1} + O(d_n^{-1}),\ \alpha \in \mathbb{R}$ as $n \to \infty$, where d_n is a sequence of positive numbers such that $nd_n \to c$ as $n \to \infty$. In this paper we investigate asymptotic behavior of the random process $X_{[nt]}^n,\ t > 0$ when $nd_n \to \infty$ as $n \to \infty$ and prove limit theorems for $X_{[nt]}^n,\ t > 0$. We remark that the obtained results are different from the results in the case $m_n = 1 + \alpha n^{-1} + o(n^{-1})$.

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