Semi-recursive nonparametric algorithms of identification and control

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Let a sequence $(Y_t)_{t=\dots,-1,0,1,\dots}$ be generated by a regression–autoregression

$$Y_t = \Psi(Y_{t-1}, X_t, U_t) + \Phi(Y_{t-1}, X_t, U_t)\xi_t, \tag{1}$$

where (ξ_t) is a sequence of zero mean i.i.d. random variables with unit variance, Y_t is an output variable, X_t, U_t are noncontrolled and controlled input random variables, not depending on (ξ_t), and Ψ and $\Phi > 0$ are unknown functions defined on $\mathbf{R}^3$.

Denote $Z_{t-1} = (Y_{t-1}, X_t, U_t)$. Note that for $x \in \mathbf{R}^3$ we have the conditional expectation $\mathbf{E}(Y_t|Z_{t-1} = x) = \Psi(x)$ and the conditional variance $\mathbf{D}(Y_t|x) = \Phi^2(x)$.

We presume that Assumptions 3.1 and 3.2 from [1] are fulfilled. Then, according to [1:Lemma 3.1], (Y_t) is a strictly stationary process, satisfying the strong mixing condition with a strong mixing coefficient $\alpha(\tau) \le c_0\rho_0^\tau$, $0 < \rho_0 < 1$, $c_0 > 0$. In this case, we can find the MSE of the proposed estimators as in [2].

We estimate $\Psi(x)$ by the statistic

$$\Psi_n(x) = \sum_{t=2}^{n+1} \frac{X_t}{h_t^3} \mathbf{K}\left(\frac{x - Z_{t-1}}{h_t}\right) \Bigg/ \sum_{t=2}^{n+1} \frac{1}{h_t^3} \mathbf{K}\left(\frac{x - Z_{t-1}}{h_t}\right), \tag{2}$$

where $\mathbf{K}(u) = \prod_{i=1}^{3} K(u_i)$ is a three-dimensional product-form kernel, $(h_n) \downarrow 0$ is a number sequence. The conditional variance for model (1) is estimated by a statistic similar to (2).

Consider the stabilization problem of Y_n on the level Y^*. Let Ψ be a simple continuous function. Then we can construct the following control algorithm for the given level Y^* :

$$U_n^* = \frac{\displaystyle\sum_{t=2}^{n+1} \frac{U_t}{h_t^2} K\left(\frac{Y^* - Y_t}{h_t}\right) K\left(\frac{Y^* - Y_{t-1}}{h_t}\right) K\left(\frac{X_{n+1} - X_t}{h_t}\right)}{\displaystyle\sum_{t=2}^{n+1} \frac{1}{h_t^2} K\left(\frac{Y^* - Y_t}{h_t}\right) K\left(\frac{Y^* - Y_{t-1}}{h_t}\right) K\left(\frac{X_{n+1} - X_t}{h_t}\right)}. \tag{3}$$

Simulations and empirical results based on the macroeconomic data of Russian Federation are provided.

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References

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