Construction of bivariate survival probability functions related to 'micro-shocks' - 'micro-damages' paradigm

Jerzy K. Filus[1] **and Lidia Z. Filus**[2]

[1] *Oakton Community College, USA, email: jfilus@oakton.edu*
[2] *Northeastern Illinois University, USA, email: L-Filus@neiu.edu*

Keywords: confidentiality, disclosure risk, Metropolis algorithm, noise multiplication, prior distribution.

In searching for a proper description of various kind of stochastic dependences among random quantities considered in reliability and biomedical investigations we apply a general method of construction of bivariate probability distributions (or the corresponding joint survival function) of such quantities. High average pulse rate and/or blood pressure, excessive level of cholesterol, or other evidently dependent in magnitude levels of some chemicals in patient's body could serve as examples of such stochastically dependent quantities.

In effort to find general device for underlying stochastic dependences among these indicators we define and employ (on the physical part) the 'micro-shocks' - 'micro-damages' pattern [3] that naturally occurs in some reliability investigations. This reliability pattern can be redefined for a wider range of phenomena such as bio-medical [1], econometric, or other "realities".

In general, we consider random variables X_1, X_2 that interact with each other so that the impact of one of them on the other is mutual in the sense that each variable is an explanatory to the other.

The joint probability distribution of each such pair can model some mutual ("physical", in a very wide sense, not only in a strict sense of the physics theory) interactions.

In the 'micro-shocks' - 'micro-damages' pattern, the realizations x_i of the random variables X_i, accordingly to their sizes influence the hazard rate (or its parameter) of the other random variable X_k, $i, k = 1, 2$ and $i \neq k$. The considered method of construction allows to obtain a joint survival function

$$S(x_1, x_2) = \mathbb{P}(X_1 > x_1, X_2 > x_2)$$

of the random vector (X_1, X_2), given both marginal survival functions $\mathbb{P}(X_1 > x_1)$ and $\mathbb{P}(X_2 > x_2)$.

It turns out that in the simplest case, when both marginals are exponentially distributed, we obtain the common first bivariate exponential Gumbel distribution [5]. In some applications one can consider the method as an extension of what we call "Gumbel device" so that any (not necessarily exponential) two marginal survival functions $\mathbb{P}(X_1 > x_1)$, $\mathbb{P}(X_2 > x_2)$ of X_1, X_2 can be "joint" by what we call "Gumbel dependence factor" $\exp(-cx_1x_2)$, where parameter c is any nonnegative real and the condition $c = 0$ stands for independence.

Realize that this construction preserves given in advance, marginal distributions. Also, one can see that the above dependence factor can be generalized to a wider class of functions. For example, one may consider the Gumbel factors in the following "Weibullian form" $\exp(-cx_1^a x_2^b)$ with positive parameters a, b.

In fact, any arbitrary two continuous marginals (not necessarily from the same class of probability distributions) may "invariantly" be "connected" by a given fixed 'Gumbel factor' to "become" stochastically dependent. In reverse, a fixed pair of marginals can be connected in many different ways each corresponding to one Gumbel factor.

Moreover, the above constructions can easily be extended to higher than two dimensions.

References

[1] Collett, D. (2003). *Modeling Survival Data in Medical Research.* Chapman&Hall, London.

[2] Filus, J. K., Filus, L. Z. (2006). On some new classes of multivariate probability distributions. *Pakistan Journal of Statistics* **22**, 21–42.

[3] Filus, J. K., Filus, L. Z. (2009). 'Microshocks-Microdamages' type of system component interaction; failure models. In: *Proceedings of the 15th ISSAT International Conference on Reliability and Quality in Design*, 24–29.

[4] Filus, J. K., Filus, L. Z., Arnold, B.C. (2010). Families of multivariate distributions involving "triangular" transformations. *Communications in Statistics - Theory and Methods* **39**(1), 107–116.

[5] Gumbel, E. J. (1960). Bivariate exponential distributions. *Journal of the American Statistical Association* **55**, 698–707.