Maximum likelihood estimates for Markov-additive processes of arrivals by aggregated data

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We consider a simplification of Markov-additive process of arrivals. Let $\mathbf{N} = \{0, 1, ...\}$, r be a positive integer, E be a countable set, and $(\mathbf{X}, J) = \{(\mathbf{X}(t), J(t)),$ $t \geq 0\}$ be a considered process on state space $N^r \times E$. The increments of $\mathbf{X}$ are associated to arrival events. Different (namely r) classes of arrivals are possible, so $X_i(t)$ = total number of arrivals in $(0, t]$ in the class i, $i = 1, 2, ..., r$. We call $\mathbf{X}$ the *arrival component* of $(\mathbf{X}, J)$, and J - the *Markov component* of $(\mathbf{X}, J)$.
Whenever the Markov component J is in the state j, the following two types of transitions in $(\mathbf{X}, J)$ may occur. 1) The i-arrivals without a change of state in $j \in E$ occur at rate $\lambda^i_j(n)$, $n > 0$. 2) Changes of state in J without arrivals occur at rate $\lambda_{j,k}$, $k \in E$, $j \neq k$.

We suppose that J is a birth and death process. Let $\overrightarrow{\lambda} = \Big(\lambda_{j,j+1} : j = 1, ..., m-1\Big)$, $\overleftarrow{\lambda} = \Big(\lambda_{j,j-1} : j = 2, ..., m\Big)$. If the state $j \in E$ is fixed, then different arrivals form independent Poisson flows. Further, let $q_i(n)$ be a probability that i-arrival contains n items, $\sum_{n>0} q_i(n) = 1$. These probabilities do not depend on state $j \in J$ and are the known ones. Now, the i-arrival rates have the following structure: $\lambda^i_j(n) = v_j\Big(\alpha^{\langle i \rangle}\Big)q_i(n)$, $j = 1, ..., m$, where v_j is a known function to an approximation of the parameters $\alpha^{\langle i \rangle} = \Big(\alpha_{1,i}, \alpha_{2,i}, ..., \alpha_{k,i}\Big)^T$.
We consider a problem of unknown parameters $\alpha = \Big(\alpha^{\langle 1 \rangle} \;\; \alpha^{\langle 2 \rangle} \;\; ... \;\; \alpha^{\langle r \rangle}\Big)_{k \times r}$, $\overrightarrow{\lambda}$ and $\overleftarrow{\lambda}$ estimation. It is supposed that we have n independent copies $X^{(1)}(t), ..., X^{(n)}(t)$ of the considered process $X(t) = \Big(X_1(t), ..., X_r(t)\Big)^T$ - total numbers of arrivals of various classes in $(0, t]$. Our initial point is the following: each $X(t)$ has multivariate normal distribution with mean $E(X(t)) = t\mu$ and covariance matrix $Cov(X(t)) = tC$, where μ is r-dimensional column vector and C is $(r \times r)$-matrix. The sample mean μ^* and the sample covariance matrix $\mathbf{C}^*$ are sufficient statistics, therefore we must make statistical inferences on this basis. In the paper maximum likelihood estimates are calculated for unknown parameters.

References

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