Modeling of financial portfolio in emerging markets

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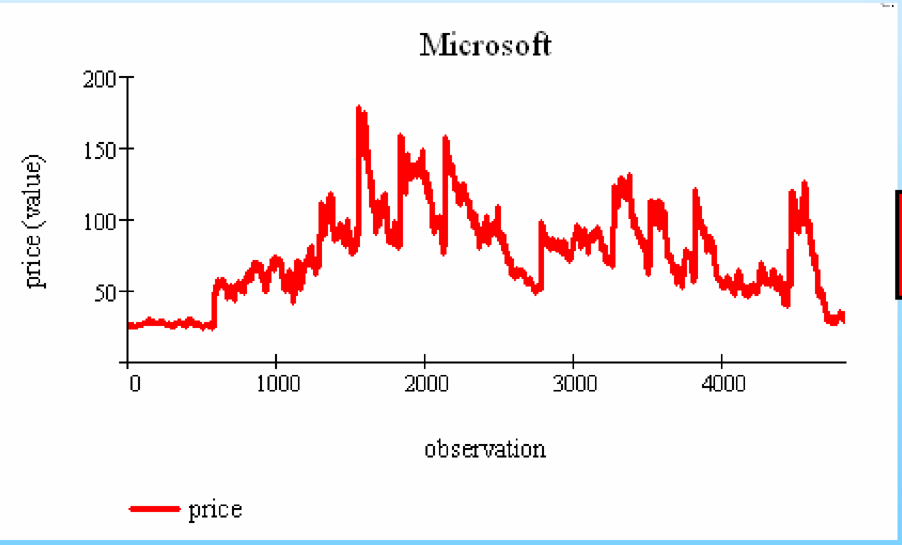
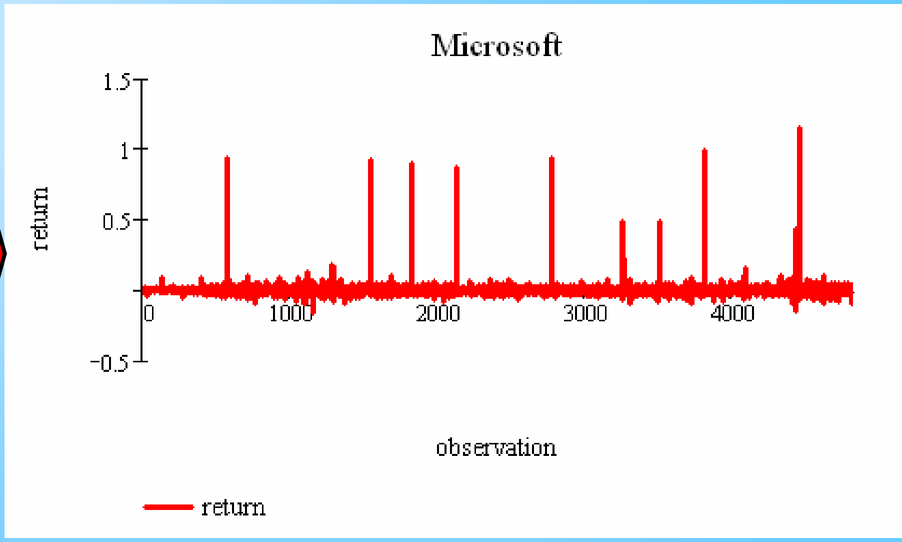
Data

P_i – observed stock price

$$X_i = \frac{P_{i+1} - P_i}{P_i}$$ - returns





Problems

Baltic States market is comparatively new (series are short 1000 – 2000 data points) and stagnation effects are often observed and expressed by an extremely strong passivity.

Daily stock return is a continuous random variable with some distribution function (Gaussian, stable etc.). But in real market, when stock price deos not changes, its return is equal to 0. When number of such observations increases, then variance tends to 0 and the distribution function becomes degenerate distribution function.

Daily return problem
(application for Baltic States equity)

We analyzed all the Baltic Main list and Baltic I-list in period 2000 – 2007. Number of daily zero stock returns for this period differs from 12% to 89% and in average is 52% !! Any distribution function fitted not to the empirical data (Anderson–Darling and Kolmogorov–Smirnov goodness–of–fit tests).

Daily return problem
(solution)

So we have presented univariate mixed-stable model and we showed that this model fits the empirical data much better than the Gaussian one.

Mixed stable model

The cumulative distribution function (CDF)

$$F_{mix}(z, p, \theta) = p \cdot \varepsilon(z) + (1 - p) \cdot F(z, \theta)$$

The characteristic function:

$$\phi_{mix}(t, p, \theta) = p + (1 - p) \cdot \phi(t, \theta)$$

The PDF of the mixed r.v. is

$$f_{mix}(x, p, \theta) = p \cdot \delta(x) + (1 - p) \cdot p(x, \theta)$$

where $\delta(x)$ is the Dirac delta function.

α-stable distribution

We say that a r.v. X is distributed by the stable law and denote

$$X \sim S_\alpha(\sigma, \beta, \mu)$$

where S_α is the probability density function.

Each stable distribution $S_\alpha(\sigma,\beta,\mu)$ has the stability index $\alpha \in (0;2]$, which can be treated as the main parameter, when we make an investment decision, $\beta \in [-1,1]$ is the parameter of asymmetry, $\sigma > 0$ is that of scale, $\mu \in \boldsymbol{R}$ is the parameter of position.

α-stable distribution

The characteristic function:

$$\phi(t)=\begin{cases} exp\left\{-\sigma^{\alpha}\cdot|t|^{\alpha}\cdot\left(1-i\beta sign(t)\tan(\frac{\pi\alpha}{2})\right)+i\mu t\right\} & \alpha\neq 1 \\ exp\left\{-\sigma\cdot|t|\cdot\left(1+i\beta sign(t)\frac{2}{\pi}\cdot\log|t|\right)+i\mu t\right\} & \alpha=1 \end{cases}$$

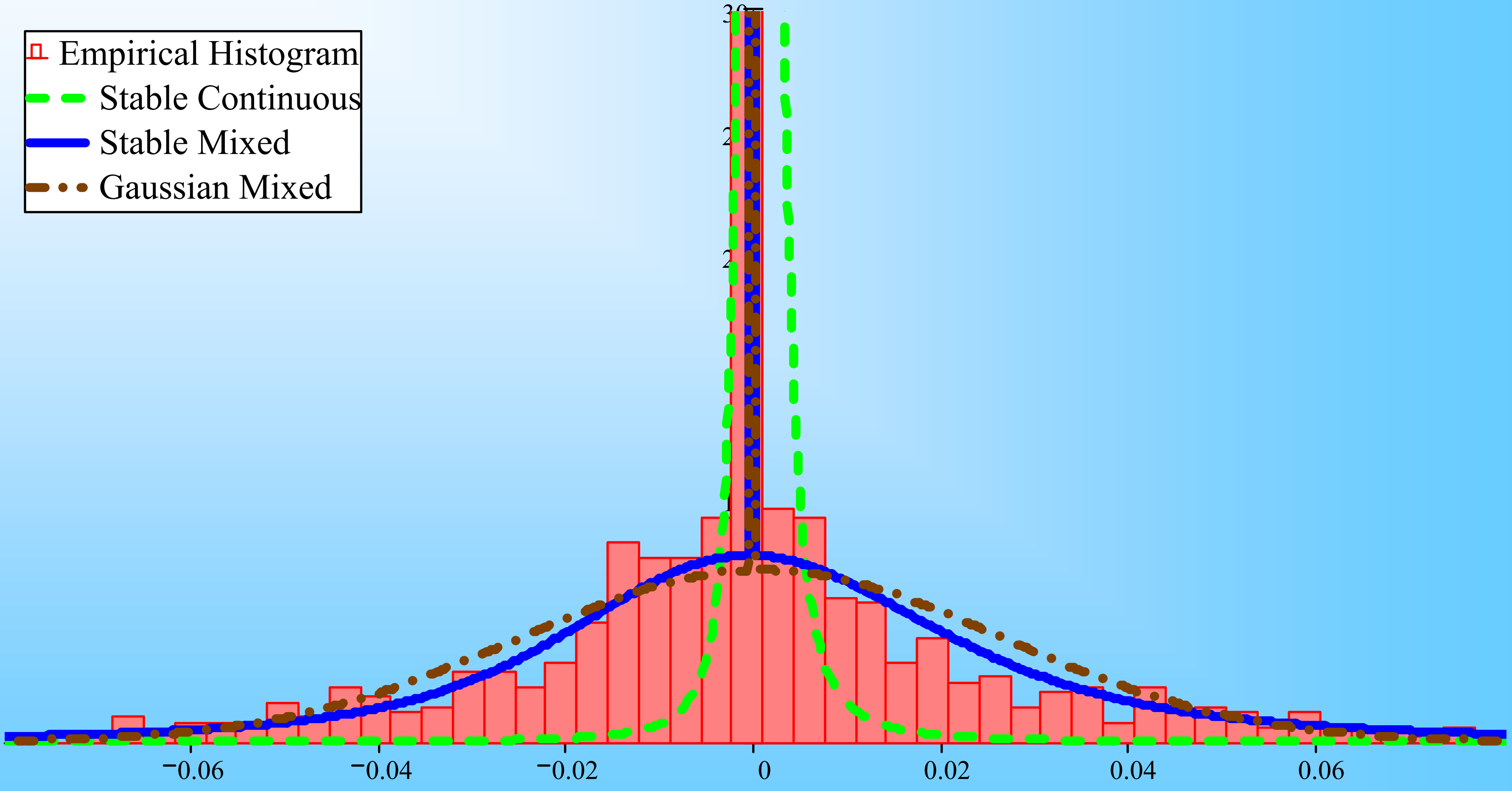
The probability density function (PDF) is

$$p_{\alpha}(x)=\int_{-\infty}^{+\infty}\phi(t)\cdot exp(ixt)dt$$

Estimates of stable parameters

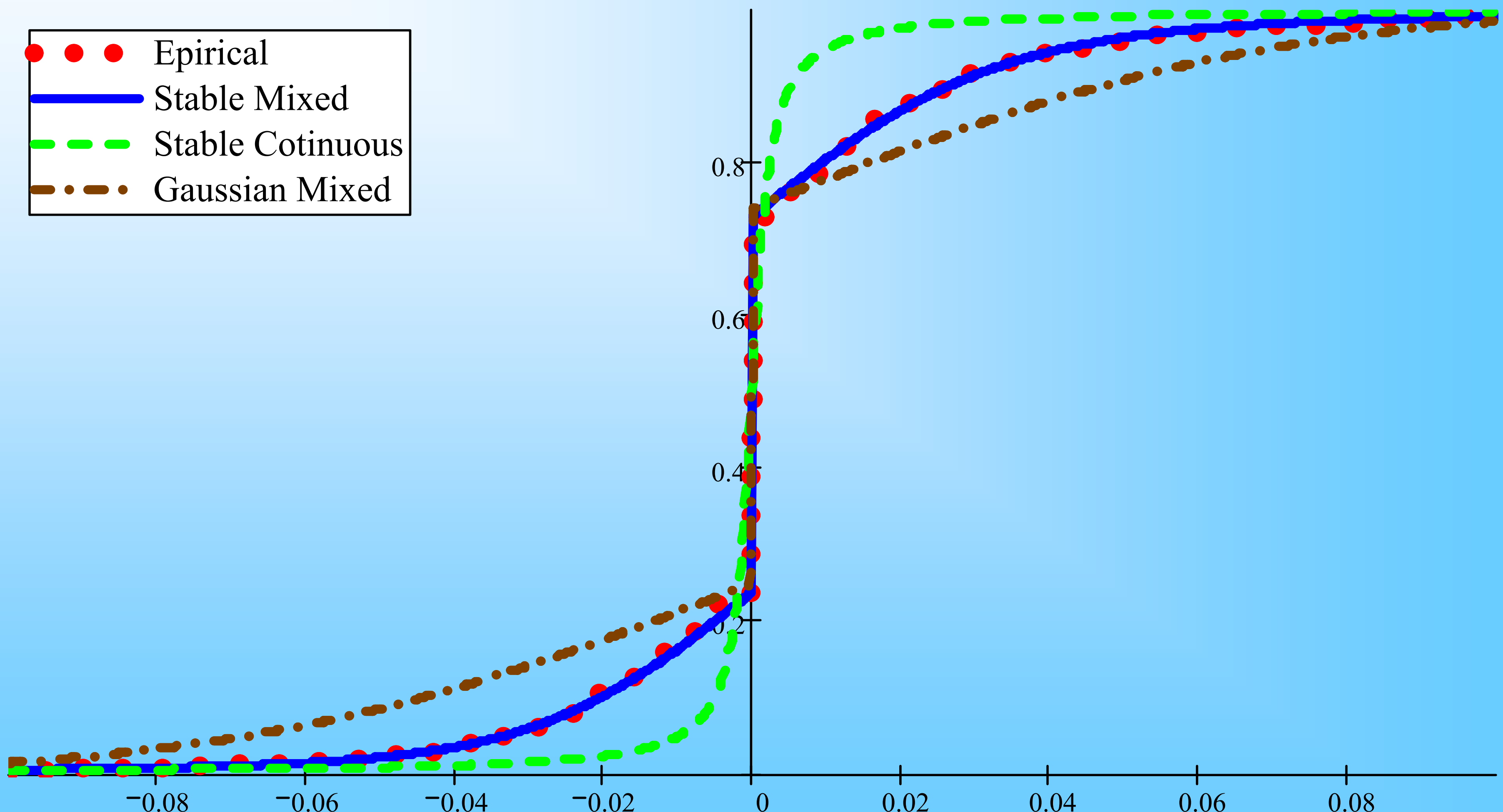
Equity	α	β	μ	σ
ETLAT	1.5309	-0.0649	0.0004	0.0067
GZE1R	1.2656	0.0559	0.0019	0.0091
LDJ1L	1.6094	0.1585	0.0019	0.0133
MKO1T	1.6313	0.1252	0.0030	0.0118
MNF1L	1.6283	0.1310	0.0020	0.0162
NRM1T	1.6158	0.0171	0.0008	0.0071
SNG1L	1.2872	0.3317	0.0059	0.0093
TEO1L	1.7832	0.0354	0.0000	0.0107
VNF1R	1.6789	0.2737	0.0028	0.0161
VNG1L	1.5384	0.1209	0.0025	0.0133

Daily return problem



Daily return problem

(solution: mixed distribution)



Relationship measures

- covariance or correlation
- rank (Spearmen, Kendall, etc.)
- contingency
- covariation and codifference

Covariation

If X_1 and X_2 are two symetric i.d. (with $\alpha_1=\alpha_2=\alpha$) stable random variables, then covariation is equal

$$[X_1, X_2]_\alpha = \int_{S_2} s_1 s_2^{\langle \alpha-1 \rangle} \Gamma(ds)$$

where $\alpha>1$, $y^{\langle \alpha \rangle} = |y|^\alpha \operatorname{sign}(\alpha)$ and Γ is a spectral measure of (X_1,X_2).

Covariation

If α=2 (Gaussian distribution) covariation is equal to half of covariance

$$[X_1, X_2]_2 = \tfrac{1}{2}\mathrm{Cov}(X_1, X_2)$$

and

$$[X_1, X_1]_2 = \sigma^2_{X_1}$$

Covariation norm of $X \in S_\alpha$ (α>1), can be calaculated as

$$\|X\| = ([X, X]_\alpha)^{1/\alpha}$$

Codifference

By definition codifference is equal

$$\mathrm{cod}_{X,Y} = \ln(E\exp i(X-Y)) - \ln(E\exp(iX)) - \ln(E\exp(-iY))$$
$$= \ln\left(\frac{E\exp i(X-Y)}{E\exp(iX)\cdot E\exp(-iY)}\right) = \ln\left(\frac{\phi_{X-Y}}{\phi_X\cdot\phi_{-Y}}\right)$$

or

$$\mathrm{cod}_{X,Y} = \ln\left(n\sum_{j=1}^{n}\exp(i(X_j - Y_j)) \cdot \left(\sum_{j=1}^{n}\exp(iX_j)\cdot\sum_{j=1}^{n}\exp(-iY_j)\right)^{-1}\right)$$

Codifference norm

In general case are proper following inequalities

$$(1-2^{\alpha-1})\ln\left(\frac{1}{E\exp(iX)\cdot E\exp(-iY)}\right)\le \mathrm{cod}_{X,Y}=\ln\left(\frac{E\exp i(X-Y)}{E\exp(iX)\cdot E\exp(-iY)}\right)\le\ln\left(\frac{1}{E\exp(iX)\cdot E\exp(-iY)}\right)$$

and, if we normalize,

$$(1-2^{\alpha-1})\le \mathrm{corr}_{X,Y}=\frac{\ln(E\exp(iX)\cdot E\exp(-iY)/E\exp i(X-Y))}{\ln(E\exp(iX)\cdot E\exp(-iY))}\le 1$$

we will get generalized correlation coefficient.

Codifference norm

If $0<\alpha \leq 1$ this correlation coefficient is only non-negative, and if $\alpha=2$, $\beta=0$, then

$$-1 \leq \text{corr}_{X,Y} = \rho_{X,Y} \leq 1$$

is equivalent to Pearson corelation coeficient.

Codifference

	TEO1L	SNG1L	VNG1L	ETLAT	GZE1R	MKO1T	NRM1T	VNF1R	LDJ1L	MNF1L
TEO1L	7.4E-04	1.1E-05	1.3E-05	6.0E-04	2.1E-05	6.0E-04	1.1E-05	3.5E-05	2.6E-05	3.0E-06
SNG1L	1.1E-05	4.2E-04	2.0E-05	1.6E-06	1.7E-05	1.6E-06	2.1E-06	5.7E-06	1.6E-06	2.9E-06
VNG1L	1.3E-05	2.0E-05	4.0E-04	8.7E-06	7.2E-05	2.2E-05	1.2E-05	4.4E-05	9.1E-06	3.9E-05
ETLAT	6.0E-04	1.6E-06	8.7E-06	1.1E-03	1.7E-05	6.0E-04	1.0E-05	2.6E-05	7.3E-06	6.9E-06
GZE1R	2.1E-05	1.7E-05	7.2E-05	1.7E-05	6.4E-04	3.3E-05	1.7E-05	4.0E-05	5.3E-06	2.6E-05
MKO1T	6.0E-04	1.6E-06	2.2E-05	6.0E-04	3.3E-05	7.2E-04	1.9E-05	1.4E-05	1.2E-05	1.2E-05
NRM1T	1.1E-05	2.1E-06	1.2E-05	1.0E-05	1.7E-05	1.9E-05	3.4E-04	2.2E-05	1.4E-05	1.9E-05
VNF1R	3.5E-05	5.7E-06	4.4E-05	2.6E-05	4.0E-05	1.4E-05	2.2E-05	2.4E-04	2.1E-06	1.2E-05
LDJ1L	2.6E-05	1.6E-06	9.1E-06	7.3E-06	5.3E-06	1.2E-05	1.4E-05	2.1E-06	5.2E-04	2.0E-05
MNF1L	3.0E-06	2.9E-06	3.9E-05	6.9E-06	2.6E-05	1.2E-05	1.9E-05	1.2E-05	2.0E-05	4.0E-04

Significance

We propose following algorithm:

- Estimate stable parameters (α, β, σ, μ) of all data sets with zeros removed;
- Construct relation (codifference of covariation) matrix ρ;
- By bootstrap method test hipothesis $\rho_{i,j}$=0:
  - With estimated parameters generate pairs of sets i and j;
  - Calculate relation measure $\hat{\rho}_{i,j}$,
  - Repeat two above steps N times (N=10000),
  - Construct ordered series;

- If $\hat{\rho}_{i,j}^{([N\cdot 0.025])} \leq \rho_{i,j} \leq \hat{\rho}_{i,j}^{([N\cdot 0.975])}$ then hypothesis $\rho_{i,j}$=0 cannot be rejected, with 0.05 significance level.

Optimal portfolio (1)

minimize,

$$\lambda \sum_{i=1}^{n} \sum_{j=1}^{n} \omega_i \omega_j \sigma_{ij} - (1-\lambda) \sum_{i}^{n} \varpi_i \mu_i$$

for all ϖ_{i}, $i = 1, \ldots, n$, with constraints

$$\sum_{i}^{n} \varpi_i = 1,$$

$$\omega_i > 0, \quad i = 1, \ldots, n$$

here ω_i weight of *i*th share, μ_i – mean of *i*th share (parameter μ), σ_{ij} – covariance between *i*th and *j*th return (codifference), λ – invertors preference (λ=0.5)

Optimal portfolio (2)

$$F(w) = c \cdot \left(\frac{1}{N} \sum_{j=1}^{N} \left| \sum_{i=1}^{n} w_i \left(X_{j,i} - \mu_i \right) \right|^{\gamma} \right)^{1/\gamma} - \sum_{i=1}^{n} w_i \mu_i$$

here γ=min(α_i), c=1/γ, μ_i – mean of ith share (parameter μ), $X_{j,i}$ – return of ith equity in jth moment.

Optimal portfolio (2)

minimize,

$F(w)$

for all $w_{\mathrm{i}}, i = 1, \ldots, n$, with constraints

$$\sum_{i}^{n} w_i = 1,$$

$$w_i > 0, \quad i = 1, \ldots, n$$

here w_i weight of ith share

Optimal solutions of systems (1) and (2)

	(1)	(2)
TEO1L	0.0193	0.000004
SNG1L	0.1475	0.2083
VNG1L	0.0652	0.0744
ETLAT	0.0231	0.0087
GZE1R	0.0639	0.2692
MKO1T	0.0085	0.1237
NRM1T	0.1359	0.0421
VNF1R	0.2816	0.1106
LDJ1L	0.1170	0.0764
MNF1L	0.1376	0.0862

System (1) gives better profit by today rate 2007.06.27

Conclusions

- Financial series are modeled by stable distributions (1,27 < α < 1,78), since covariance does not exist, it was replaced by codifference.
- Relation measures were applied to construct optimal portfolio of 10 equities from Baltic States market.
- We show that the codifference application strongly simplifies the construction of the optimal portfolio.

Thanks for listening

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