## Estonian Statistical Society 20

## Optimum pellet-group count design for estimating the size of the Swedish elk and roe deer populations

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## Pellet-group inventory

- Systematic pellet-group counts in the U.S. originated in the late 1930's. The first general overview of the method was given by Neff (1968).
- In Sweden, the first pellet-group count inventories date back to the 1970ies.
- Field inventories using the pellet-group count technique are carried out annually in Sweden. Pellet-group counting is performed using the NILS-tracts.
- Main purpose: to obtain information about fluctuations in the elk and roe deer populations. The information is needed for a sustainable management of the populations.


## NILS-tracts (national inventory of the Swedish landscape)

NILS-tracts: 631 permanent sample plots systematically distributed over whole Sweden.


## Tract layout



## Optimum design for pellet-group counting inventory

- Both statistical and cost-benefit aspects have to be considered.
- Tract size and layout should correspond to one working day for an observer.
- The main statistical criterion: minimize the variance of the estimated number of pellet groups per area unit for the studied region. Then it is important to minimize the variance for the tract given that the workload is one day.
- To find an efficient design, we can vary
- the number and layout of elk/roe deer plots;
- the length of the tract side and the number of tracts.
- Guidelines can be given based on both theoretical and empirical considerations.


## Index measure of population density

- Let $y_{i, j}=$ number of pellet groups in plot $j$ of tract $i$,

$$
x_{i, j}=\left\{\begin{array}{l}
1, \text { if counting has been performed for plot } j \text { of tract } i, \\
0, \text { else. }
\end{array}\right.
$$

- Estimate the mean number of pellet groups per study plot and its mean squared error (MSE for ratio estimator):

$$
\hat{k}=\frac{\sum_{i, j} y_{i, j}}{\sum_{i, j} x_{i, j}}, \quad \widehat{\operatorname{MSE}}_{\hat{k}} .
$$

- A single estimate $\hat{k}$ is not so informative, since it is an index that measures population density indirectly. It becomes meaningful when the estimates can be compared for several years.


## Index estimates for Värmland

| Elk | $\mathrm{n}=40$ |  | $\mathrm{n}=20$ |  |
| :--- | :---: | :---: | :---: | :---: |
|  | $\hat{k}$ | Root $\widehat{\mathrm{MSE}}_{\hat{k}}$ | $\hat{k}$ | Root $\widehat{\mathrm{MSE}}_{\hat{k}}$ |
| $2008(29)$ | 0.37 | $13 \%$ | 0.35 | $16 \%$ |
| $2009(32)$ | 0.42 | $10 \%$ | 0.43 | $11 \%$ |
| $2010(28)$ |  |  | 0.49 | $21 \%$ |


| Roe deer | $\mathrm{n}=40$ |  | $\mathrm{n}=20$ |  |
| :---: | :---: | :---: | :---: | :---: |
|  | $\hat{k}$ | Root $\widehat{\mathrm{MSE}}_{\hat{k}}$ | $\hat{k}$ | Root $\widehat{\mathrm{MSE}}_{\hat{k}}$ |
| $2008(29)$ | 0.045 | $23 \%$ | 0.056 | $27 \%$ |
| $2009(32)$ | 0.050 | $21 \%$ | 0.047 | $22 \%$ |
| $2010(28)$ | 0.031 | $30 \%$ | 0.028 | $39 \%$ |

## Elk: correlation estimates for Värmland

n - the number of study plots per tract
d - the distance between the plots in 100 meters

|  | $n$ | $d=1$ | $d=2$ | $d=3$ | $d=4$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $2008(29)$ | 40 | 0.20 | 0.12 | 0.03 | 0.09 |
|  | 20 |  | 0.25 |  | 0.10 |
| $2009(32)$ | 40 | 0.21 | 0.08 | 0.07 | 0.05 |
|  | 20 |  | 0.05 |  | 0.04 |
| $2010(28)$ | 20 |  | 0.24 |  | 0.08 |

## Theoretical correlation functions

- Suppose that $n$ study plots are located equidistantly along the perimeter of a square tract with side length of 1 km . Consider exponential correlation functions

$$
C(d)=e^{-h d}
$$

h - determines the steepness of the correlation function, d - distance.

- $\operatorname{Var}(\hat{k})$ for different number of study plots and different parameter values:

| n | $\mathrm{h}=1$ | $\mathrm{~h}=11$ | $\mathrm{~h}=16$ | $\mathrm{~h}=32$ |
| :---: | :---: | :---: | :---: | :---: |
| 20 | 0.5104 | 0.0634 | 0.0545 | 0.0502 |
| 28 | 0.5111 | 0.0559 | 0.0442 | 0.0365 |
| 40 | 0.5115 | 0.0518 | 0.0383 | 0.0272 |

## Comparing tract layouts

- Example. Let the correlation function $C(d)$ be given by

$$
C(d)=0.6 e^{-11 d}+0.4 e^{-16 d}
$$

- Consider the following tract designs:
(a) tract side length of $1.4 \mathrm{~km}, 28$ plots, 30 tracts;
(b) tract side length of $1 \mathrm{~km}, 40$ plots, ? tracts.
- How many tracts do we need in case (b) to obtain the same precision as in (a)?
- We need 33 tracts, because

$$
\frac{\operatorname{Var}_{(b)}(\hat{k})}{\operatorname{Var}_{(a)}(\hat{k})}=\frac{0.0464}{0.0426} \approx \frac{33}{30}
$$

## Grimsö, roe deer plots



Year 2000, Root $\widehat{\mathrm{MSE}}_{\hat{k}}$ when clusters considered as independent observations:

| CNESW | NESW | NCS | WCE | NS | WE | Single plot |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $9 \%$ | $10 \%$ | $10 \%$ | $10 \%$ | $11 \%$ | $11 \%$ | $14-16 \%$ |

## Comparison of cluster layouts

- Correlation function: $C(d)=\rho^{d}$
- Compare $\operatorname{Var}\left(\bar{z}_{5}\right)$ with $\operatorname{Var}\left(\bar{z}_{4}\right)$, where $\bar{z}_{5}$ and $\bar{z}_{4}$ are the means of 5 and 4 study plots.

5 plots versus 4 plots


## Suggested tract layout

28 elk plots, 200 m between the plots
$28 \times 3=84$ small roe deer plots



## References

Neff, D.J. (1968). Review of pellet-group count technique, Journal of Wildlife Management, 32.

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