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# A General Surplus Decomposition Principle in Life Insurance


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▶ Overarching decomposition principle is missing

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**Goal:** Find adapted processes $D_1, \ldots, D_m$ that start at zero with

$$\boxed{R(t) - R(0) = D_1(t) + \cdots + D_m(t)}$$

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&\quad + \sum_{t_l, t_{l+1} \in \mathcal{T}} \Big( U(t_{l+1}, t_{l+1}, t_l, \ldots, t_l) - U(t_{l+1}, t_l, \ldots, t_l) \Big) \\
&\quad + \cdots \\
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- Transition to a sequence of partitions with vanishing step lengths

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The random vector $D(t) = (D_1(t), \ldots, D_m(t))$ that satisfies

$$D_i(t) = \operatorname*{plim}_{n\to\infty} D_i^n(t)$$

is called *ISU (infinitesimal sequential updating) decomposition* of $R(t) = \varrho(X^t)$ with respect to $(\mathcal{T}_n)_n$.

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- Insurance cash flow $B$ with

$$\mathrm{d}B(t) = \sum_j I_j(t-)\,\mathrm{d}B_j(t) + \sum_{jk:j\neq k} b_{jk}(t)\,\mathrm{d}N_{jk}(t)$$

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where

$$\mathbb{E}^* \left[ \int_t^T \frac{\kappa^*(t)}{\kappa^*(s)} \mathrm{d}B(s) \middle| Z(t) = j \right] \text{ and } \mathrm{d}\kappa(t) = \kappa(t-)\mathrm{d}\Phi(t), \ \kappa(0) = 1.$$

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Proposition: It holds

$$R(t) = -H((\Phi^*, \Lambda^*) + (\Phi - \Phi^*, N - \Lambda^*)^t),$$

where for any valuation basis $(\overline{\Phi}, \overline{\Lambda})$ the mapping $H$ is defined by

$$H((\overline{\Phi}, \overline{\Lambda})) := \sum_j \int_{[0,T]} \frac{1}{\overline{\kappa}(s)} \overline{p}_{aj}(0, s-) \mathrm{d}B_j(s) + \sum_{j,k:j \neq k} \int_{(0,T]} \frac{1}{\overline{\kappa}(s)} \overline{p}_{aj}(0, s-) b_{jk}(s) d\overline{\Lambda}_{jk}(s)$$

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In case 2), we obtain the ISU decomposition

$$D_u(t) = -\sum_{jk:j\neq k} \int_{(0,t]} \frac{1}{\kappa(s)} I_j(s-) R^*_{jk}(s) \mathrm{d}(N_{jk} - \Lambda_{jk})(s),$$

$$D_j(t) = \int_{(0,t]} \frac{1}{\kappa(s-)} I_j(s-) \Big( V^*_j(s-) \mathrm{d}(\widetilde{\Phi} - \Phi^*)(s) - \sum_{k:k\neq j} R^*_{jk}(s) \mathrm{d}(\Lambda_{jk} - \Lambda^*_{jk})(s) \Big),$$

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As a special case, the ISU decomposition includes heuristic approaches of Ramlau-Hansen (1988) and Norberg (1999).

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Splitting the financial risk into an unsystematic and a systematic part, one can replicate the surplus formula of Asmussen & Steffensen (2020).

# Further results

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The mean portfolio (revaluation) surplus is given by (cf. Norberg, 1999)

$$\overline{R}(t) = \mathbb{E}\left[ -\int_{[0,t]} \frac{1}{\kappa(s)} \mathrm{d}B(s) - \sum_j \frac{1}{\kappa(t)} I_j(t) V_j^*(t) \middle| \Phi, \Lambda \right],$$

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ISU decomposition yields again traditional decomposition formulas by Ramlau-Hansen (1991) and Norberg (1999).

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**THANK YOU FOR YOUR ATTENTION!**

# Literature I

Asmussen S., Steffensen M. (2020). *Risk and Insurance*, Vol. 96 of Probability Theory and Stochastic Modeling. Switzerland: Springer.

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# Alternative decomposition principles - One-at-a-time

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Drawback: "Joint risk factor" cannot be assigned to any source of risk

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$$D_m(t) = \sum_{t_l, t_{l+1} \in \mathcal{T}} \Big( U(t_l, \dots, t_l, t_{l+1}) - U(t_l, \dots, t_l) \Big),$$

$$\overline{D}(t) = R(t) - R(0) - \sum_{j=1}^{m} D_j(t)$$

is called the *OAT (one-at-a-time) decomposition* of $R(t) - R(0)$ w.r.t. $\mathcal{T}$. (cf. Biewen, 2014)

Drawback: "Joint risk factor" cannot be assigned to any source of risk

- Transition to a sequence of partitions with vanishing step lengths *(IOAT decomposition)*

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The random vector $D(t) = (D_1(t), \dots, D_m(t))$ defined by

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$$\dots$$

$$D_m(t) = \frac{1}{m!} \sum_{\pi \in \sigma_m} D^{\pi}_{\pi(m)}(t),$$

is called the *averaged ISU decomposition* of $R(t) - R(0)$ w.r.t. $(\mathcal{T}_n(t))_n$.
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Theorem: If the ISU decomposition is independent of update order, then ISU (for each update order), IOAT and averaged ISU yield the same decomposition.